Popper's test of quantum mechanics and two-photon "ghost" diffraction

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A test on quantum mechanics proposed long ago by Karl Popper is reconsidered with further detail and new insight. An ambiguity in the proposal, which turns out to be essential in order to make the test conclusive, is identified and taken into account for the first time. Its implications for recently performed photon experiments [such as in D. V. Strekalov *et al.*, Phys. Rev. Lett. **74**, 3600 (1995)] are briefly analyzed.

Quantum Mechanics (QM) plays an essential role in our present understanding of many fundamental physical phenomena. Most of them manifest highly counterintuitive behaviours and the corresponding QM explanation is full of subtle and unfamiliar concepts. As a consequence, misconceptions and controversies have been — and continue to be— quite long and frequent. An excellent example is the thought experiment that Karl Popper proposed and continued to improve between 1934 and 1987 [1, 2, 3, 4]. In the most recent versions, Popper makes a twofold assertion: *i)* that his is a *crucial* test [2, 4] on the interpretation of QM and *ii)* that he "is inclined to predict" that, if experimentally performed, the test would decide against the (subjectivist) Copenhagen interpretation [2]. Needless to say, such statements have raised a considerable and interesting controversy [5, 6, 7, 8, 9, 10, 11, 12, 13], both on the conclusive character of the test and on its relation with recently performed experiments on two-photon "ghost" diffraction [14, 15].

The purpose of our paper is to contribute to clarify these issues: *i)* by identifying an ambiguity in Popper's proposal which, in spite of being essential in order to obtain the correct QM predictions, it has not been explicitly considered in previous analyses and *ii)* by establishing the connection between Popper's test and the two-photon diffraction experiment by Strekalov *et al.* [15].

The experimental setup proposed by Popper is quite simple. Its projection on the plane comprising the horizontal x-axis and the vertical y-axis is schematically represented in Fig. 1. Two vertical and parallel screens are placed on the planes $x = \pm d$ left and right from the origin, Fig. 1a. There is a slit on each screen which allows the passage of photons with vertical coordinates in a narrow range, $-a \leq y \leq +a$. Photons are emitted in pairs from a source S placed at the origin —positronium decays into two γ's, in Popper's original proposal— and the analysis restricts to photon pairs whose members are both jointly detected. Some pairs will indeed have both members passing through the slits and the vertical momenta of these left- and right- moving photons, $(k_1)_y \equiv k_1$ and $(k_2)_y \equiv k_2$, will finally be measured by two arrays of detectors placed at distances D far away behind the slits, $D \gg a$. For suitable slit widths and photon momenta, $|\vec{k}_{1,2}| \sim \pi/a$, single-slit diffraction theory predicts $\Delta k_{1,2} \simeq 1/2a$ for the dispersions of the vertical components of these momenta.

Popper further assumes that the source decays at rest and the two photons are thus emitted with opposite momenta, $\vec{k_1} + \vec{k_2} = 0$. Thanks to this momentum entanglement between the two members of each pair, one can analyze what happens in a second run of the experiment when the right-side slit is wide open or, in practice, the right-screen removed, Fig. 1b. Popper concludes that, for coincident detections, standard QM predicts that the right-side momentum dispersion, $\Delta k_2 \simeq 1/2a$, remains and can be further increased by narrowing the width $2a$ of the still *in situ* left-slit. This prediction follows from the assumption that the passage of the left-moving photon through the left-slit amounts to performing a measurement of the y-coordinate with $\Delta y_1 \simeq a$ and that, as a consequence of the entanglement and common origin of the two photons, the vertical position of its right-moving partner is known with the same accuracy $\Delta y_2 \simeq a$. Popper is quite explicit on this point [2]:

> To sum up: if the Copenhagen interpretation is correct, then any increase in the precision of our *mere knowledge* of the position [y_2] of the particles going to the right should increase their scatter; and this prediction should be testable.

In other words, for coincident detections a "ghost" image of the left-slit performs a kind of "indirect measurement" of y_2 and produces "virtual" diffraction on the right-side. The momentum dispersion is predicted to be $\Delta k_2 \simeq 1/2a$ by considering that, according to standard QM, Heisenberg's principle is "applicable to all kinds of indirect measurements" [2]. By contrast, Popper's objective interpretation of QM —based on propensities rather than on subjective knowledge— predicts a decrease of Δk_2 when the left-side slit is narrowed. This makes the test a *crucial* one [2].

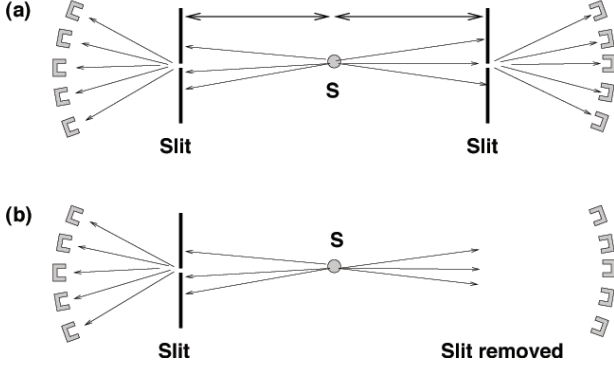A serious criticism to the preceding analysis has been

FIG. 1: Schematic diagram of Popper's proposal. (a) Two symmetrical narrow slits produce similar single-slit diffraction patterns for two-photon events detected in coincidence by the left- and right-arrays of detectors. (b) The slit on the right is wide open (the screen has been removed) and the right-side diffraction pattern is analyzed for coincident events as in (a).

pointed out by several authors [6, 7, 8, 9, 10, 11, 12, 13]. The source S cannot be exactly localized at the origin and perfectly at rest, as required to argue that, for jointly detected events, one has $\Delta y_1 = \Delta y_2 \simeq a$ regardless the presence (Fig. 1a) or not (Fig. 1b) of the right-slit. The undecayed source itself or, equivalently, the global two-photon final system has to obey Heisenberg's principle and, accordingly, the vertical components of the CM-position, $(y_1 + y_2)/2$, and total momentum, $k_1 + k_2$, must satisfy $\Delta(k_1 + k_2)\Delta[(y_1 + y_2)/2] \geq 1/2$. Once this constraint is imposed, the analyses of Refs. [6, 7] —based on simple geometrical arguments— claim that Popper's proposal is no longer able to discriminate between the two approaches. Similarly, the discussion in [11, 12] —based on a two-particle state compatible with Heisenberg's principle and a simplified assumption on wave-function reduction— claims that in the case of Fig. 1b standard QM predicts no increase of Δk_2 when narrowing the left-side slit. A claim which contradicts Popper's original analysis [2, 4] and would make his test inconclusive as well.

In spite of all these claims, we have recently argued [16] that the above criticisms —which are fully reasonable, at least in principle— do not necessarily destroy the whole "falsifiability" potential of Popper's proposal. The situation is more subtle and deserves further analysis [8]. There is an ambiguity in Popper's proposal —a new feature which has not been considered in previous analyses— that plays a central role. In our view, the identification and resolution of this ambiguity clarifies the whole issue, as we now proceed to discuss.

As in Ref. [16], let's consider the two-photon entangled wave-function

$$\Psi(y_1, y_2) = \int\int dk_1 dk_2 \Psi(k_1, k_2) \frac{e^{ik_1 y_1}}{\sqrt{2\pi}} \frac{e^{ik_2 y_2}}{\sqrt{2\pi}} , \quad (1)$$

where only the relevant, vertical components appear explicitly, $\hbar = 1$, the integration limits are $\pm\infty$, and

$$\Psi(k_1, k_2) = \frac{1}{\sqrt{\pi\sigma_+\sigma_-}} e^{-\frac{1}{4\sigma_+^2}(k_1+k_2)^2} e^{-\frac{1}{4\sigma_-^2}(k_1-k_2)^2} . \quad (2)$$

State (2) can be Fourier-transformed into

$$\Psi(y_1, k_2) = \sqrt{\frac{2}{\pi} \frac{\sigma_+\sigma_-}{\sigma_+^2 + \sigma_-^2}} e^{-\frac{\sigma_+^2\sigma_-^2 y_1^2 + k_2^2 - i(\sigma_+^2 - \sigma_-^2)y_1 k_2}{\sigma_+^2 + \sigma_-^2}} , \quad (3)$$

and

$$\Psi(y_1, y_2) = \sqrt{\frac{\sigma_+\sigma_-}{\pi}} e^{-\frac{1}{4}[\sigma_+^2(y_1+y_2)^2 + \sigma_-^2(y_1-y_2)^2]} . \quad (4)$$

Note that for the global system we have chosen a Gaussian wave packet with $\Delta(k_1 + k_2) = \sigma_+$ and $\Delta[(y_1 + y_2)/2] = 1/2\sigma_+$. This allows for analytical computations and one has $\Delta(k_1 + k_2)\Delta[(y_1 + y_2)/2] = 1/2$, which is the minimum value compatible with Heisenberg's principle. In this sense, our state is the closest analog to Popper's proposal, which naively assumed a perfectly fixed and localized source. It also coincides with the state discussed by Qureshi [12, 13] once σ_+^2 and σ_-^2 are substituted by $1/\Omega_0^2$ and $4\sigma^2$. A Gaussian wave packet is also quite appropriate to discuss the "ghost" diffraction experiments [14, 15], where an entangled two-photon state is obtained by spontaneous parametric down conversion (SPDC); indeed, the pump laser which plays the role of the decaying system has an approximate Gaussian profile. Although the two SPD converted photons do not travel with opposite momenta $(\vec{k_1} + \vec{k_2} = \vec{k}_{\text{pump}})$, an "unfolded" version of the experimental scheme (see Refs. [14, 15]) is known to be equivalent to the schemes in Fig. 1. On the other hand, the width of the pump beam is so large (a few mm) and its divergence so small (less than a mrad) that the SPD converted two-photon states in Refs. [14, 15] can be modelled by the state (1) with a large $\Delta[(y_1 + y_2)/2]$ and a small $\sigma_+ \ll \sigma_-$.

We thus assume that the state (1) describes our system when the left-moving photon reaches the screen in the setups of Fig. 1 [17]. Since we are going to restrict the discussion to jointly detected photon pairs and this requires the passage of the left-photon through the slit, previous analyses (including Popper's) consider that this effective passage through the slit amounts to performing a *measurement* of the y-coordinate of the left-moving photon with an accuracy $\Delta y_1 \simeq a$. In our view, this is not exactly the case and generates important ambiguities. Strictly speaking, the effective passage through the left-slit amounts to perform a *state preparation* for the *global* system one is going to work with. The two photons remain entangled and none has a (pure) quantum state by itself until a measurement (including photon detection) is really performed. But there are many possibilities for such a measurement and, somehow unexpectedly,

they lead to inequivalent situations. We concentrate on the two more interesting cases of measurement: central *versus* inclusive detections.

Central detection: One possibility consists in collecting coincident events with the left photon reaching the *central* detector on the negative x-axis far left from the slit. This amounts to select left-photons with a measured $k_1 = 0$. When one of these photons passes through the $2a$-wide slit, its y-coordinate is established with an accuracy $\Delta y_1 \simeq a$. Then the indistinguishable photon paths covering the whole allowed range, $-a \leq y \leq +a$, reach the central detector in phase and the corresponding amplitudes add coherently thus building the zero-th order diffraction maximum. Once the left-moving photon is so detected, *and only then*, its corresponding right-partner has a definite, pure quantum state and QM can be used to make unambiguous and complete predictions for measurements performed on it.

In particular, for the vertical momentum of these photons QM predicts a vanishing expectation value, $\langle k_2 \rangle = 0$, and a variance $(\Delta k_2)^2$ given by

$$(\Delta k_2)^2_a|_{CD} = \frac{\int_{-\infty}^{+\infty} dk_2 k_2^2 |\int_{-a}^{+a} dy_1 \Psi(y_1; k_2)|^2}{\int_{-\infty}^{+\infty} dk_2 |\int_{-a}^{+a} dy_1 \Psi(y_1; k_2)|^2} \ , \quad (5)$$

with integrals covering the opening of the left-slit and introducing the a-dependence we are looking for. For reasonably narrow slit-widths one can expand the Gaussian functions and retain the first three terms to obtain

$$(\Delta k_2)_a|_{CD} = \frac{1}{2}\sqrt{\sigma_+^2 + \sigma_-^2}\left[1 - \frac{a^2}{12}\frac{(\sigma_+^2 - \sigma_-^2)^2}{(\sigma_+^2 + \sigma_-^2)}\right] \ . \quad (6)$$

Therefore, QM predicts an increase in the momentum spread of the right-moving photon when narrowing the left-slit width, which is precisely the same conclusion that Popper himself derived from standard QM. Since there are no reasons to modify Popper's conclusions about the opposite behaviour predicted by his own approach, we agree with Popper in that the test is *crucial*. At least, if the experiment is performed as just described.

And, as far as we know, present day experiments related to Popper's test are performed in this way. One is the experiment by Kim and Shih [14], which explicitly claims to be a realization of Popper's proposal. The other is due to Strekalov *et al.* [15] and makes no such a claim, but has been recently interpreted by Qureshi [13] as a real Popper's test as well. In both experiments, pairs of optical photons are jointly detected. One member of each pair (corresponding to our left-moving photon) triggers a *fixed* detector centered on the optical axis, while the momentum of its partner (our right-moving photon) is measured by a second detector scanning the region covered by the right-array of detectors in Figs. 1a,b. Our discussion should thus apply to these experimental setups and the meaning of their results will be commented after considering a second detection possibility.

Inclusive detection: Alternatively, the final left-photon detection could be performed not only by the central detector, as before, but by the whole array of left-side detectors. One thus collects a larger set of jointly detected events. For each one of these events, a measurement of the vertical momenta of both members, k_1 and k_2, has been performed. Symmetry considerations and QM still predict the same vanishing $\langle k_2 \rangle = 0$ but the right-photon momentum spread Δk_2 turns out be different from the previous one, Eq. (6). It can be computed by noticing that one has to sum over all the k_1-values measured for the photons passing through the left-slit and that this is equivalent to compute

$$(\Delta k_2)^2_a|_{ID} = \frac{\int_{-\infty}^{+\infty} dk_2 k_2^2 \int_{-a}^{+a} dy_1 |\Psi(y_1; k_2)|^2}{\int_{-\infty}^{+\infty} dk_2 \int_{-a}^{+a} dy_1 |\Psi(y_1; k_2)|^2} \ , \quad (7)$$

because the measurement one chooses to perform on the left-photon (the precise vertical position y_1 when passing through the slit, Eq. (7), rather than its vertical momentum k_1 when reaching a detector) cannot modify the (one side) probability distributions (and thus the value for Δk_2) of the right-moving photons. Eqs. (3) and (7) lead immediately to

$$(\Delta k_2)_a|_{ID} = \frac{1}{2}\sqrt{\sigma_+^2 + \sigma_-^2} \ , \quad (8)$$

with *no* a-dependence and contrasting again with the predictions of Popper's approach. The test is still *crucial*.

We have thus identified an ambiguity in Popper's proposal which has to be solved in order to obtain the correct QM prediction on the behaviour of Δk_2 when narrowing a. Δk_2 is predicted to increase for right-detections conditioned to the outcome $k_1 = 0$ corresponding to a 'click' of the *central* detector on the left. But for coincidence detections with some other (asymmetrically placed) left-detectors ($k_1 \neq 0$), Δk_2 has to decrease and Popper's test is no longer conclusive. The two opposite behaviours compensate and Δk_2 is predicted to be a-independent for *inclusive* detection over the whole left-array, *i.e.*, when one decides to ignore the existing information on the left-photon momenta.

An important question is to decide which scheme — with central or inclusive detection— is closer to Popper's original proposal. But there seems to be no clear answer for this question. The main reason has been anticipated by Qureshi [13]. The central point in Popper's argument is that the accuracy of the knowledge that one can have on the photon vertical coordinates is fixed by the (left) slit-width, $\Delta y_1 \simeq a$. As long as the set of collected photons (on the right-side) are those whose other partner passed through the (left) slit, they should show the effects that Popper was looking for. But this is a common feature in both detection schemes and none can thus be favoured. One could argue that the schematic diagrams in Popper's writings show symmetrical arrays of detectors on both sides and that their purpose, apart from

exhibiting diffraction by the physical slits in the setup of Fig. 1a, is also to collect all the photons crossing the slit in Fig. 1b. This could be true, but the fact that in this case one has to ignore the information obtained on the left-photon momenta disfavours this interpretation of Popper's argument, an argument centered precisely on exploiting the available information. In our view, between the two considered options —a first one using all the information but not the full set of events *versus* a second one using all the events but not the full information— none is clearly favoured.

Let's finally discuss the available experimental information coming from Refs. [14, 15] where, as previously mentioned, photons passing through the slit are collected only by the central detector and has thus to be analyzed in terms of Eq. (6). Unfortunately, in the experimental realization of Popper's test by Kim and Shih [14] the width of the slit is kept fix, $2a = 0.16$ mm. This precludes any direct conclusion via Eq. (6). Note, however, that the slit width is much narrower than the incident beam and that the use of such an extended source (a wide beam) is a way to respect Heisenberg's principle for the source and to circumvent the conventional criticisms previously mentioned. The experimental findings in Ref. [14] have been clearly interpreted by Short [10] but they are of marginal relevance for our present discussion. By contrast, the experiment by Strekalov *et al.* [15] is more interesting because the slit-width is modified (from 0.1 and 1.1 mm) and this allows to discriminate between the a-dependence predicted by standard QM, Eq. (6), and the opposite behaviour attributed by Popper to his own approach. The data of Ref. [15] clearly show that Δk_2 increases when a is decreased and are compatible with the QM prediction in Eq. (6). Contrary to what Popper was "inclined to predict", this experiment contradicts his own approach and vindicates conventional QM.

The same conclusion has recently been reached by Qureshi [13] and was expected by many authors (see, *e.g.* [5, 16]). But our present discussion shows that these claims were not well justified. Admittedly, they have turned out to be correct, but this is a consequence of the set of photon pairs —conditioned to central left-detection— used in Ref. [15]. For some other non-centered detectors, coincident events should show the opposite behaviour: Δk_2 increasing with a, as in Popper's approach but without contradicting QM —as previous analyses would suggest. Finally, if the set is enlarged to include all the events reaching whatever detector behind the (left) slit, QM predicts that Δk_2 is a-independent, Eq. (8), contradicting Popper's approach and making the test conclusive again. There seems to be no data for this case and an experiment will be welcome. After all, Popper's test and QM itself are really subtle, as seen in the present paper and as brilliantly emphasized by Asher Peres [8].

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[17] There is no need to consider explicitly the time evolution of our state from the source up to the screens, as done in Refs. [13] and [16] for massive particles, thanks to the tiny divergence of the beams.